Ultimate State

MEM 355 Performance Enhancement of Dynamical Systems

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Outline

- Brief Review
- Design Criteria – two step process
  - Ultimate state error (this lecture)
  - Transient response (following lectures)
- Ultimate Error Response to Command ~ Unity Feedback Systems
- Ultimate Error Response to Command ~ General Case
- Example
- Error Response to Disturbance

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Brief Review

- Cruise Control Example
  - Introductory example, illustrating basic concepts & objectives of feedback control
- Introduction of 3 essential transfer functions of generic closed loop
  - Sensitivity function (command to Error), Complementary sensitivity function (command to output), & command to control
- Bode and Nyquist Plots
  - Robustness measures, gain and phase margins
  - Sensitivity function peak – implications for robustness, reduced damping ratio, peak overshoot

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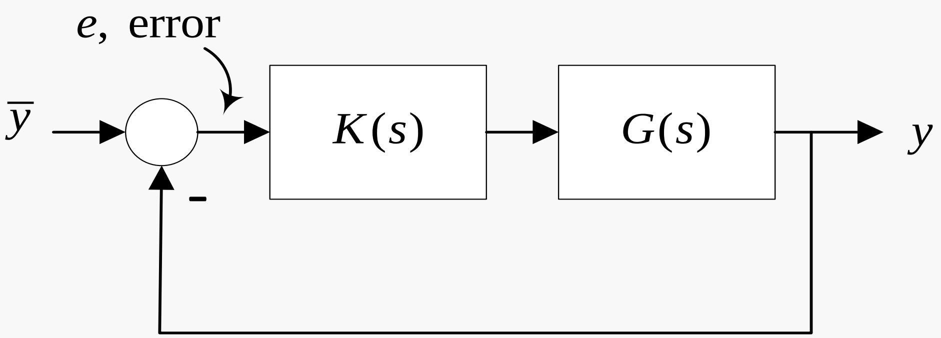
Control System Performance

- Command tracking
  - Match output to changing command (minimize/eliminate error)
  - Shape transient response (overshoot, damping)
- Disturbance rejection
  - Minimize/eliminate output error while system is subjected to disturbance
  - Shape transient response (overshoot, damping)
- Stabilization
  - Insure adequate degree of stability (decay rate, damping, pole locations)
  - Insure stability robustness (gain & phase margins)
- Handling qualities
  - From perspective of operator (pilot, driver) – maneuverability, stability
- Reliability, Safety – emerging theory

Unity Feedback Systems



$L(s) \triangleq G(s)K(s)$ open loop transfer function

$Y(s) = \dfrac{L(s)}{1+L(s)}\bar{Y}(s)$ output response transfer function (closed loop)

$E(s) = \dfrac{1}{1+L(s)}\bar{Y}(s)$ error response transfer function (closed loop)

$e(t) = \mathcal{L}^{-1}\left[E(s)\right]$, then plug in $t = \infty$ to get $e(\infty)$

or, much easier, use Final Value Theorem: $\boxed{e(\infty) = \lim_{s\to 0} sE(s)}$

Error Response to Polynomials

Test inputs: polynomial in t, step, ramp, parabola, ...

$$\text{step: } u(t) \to 1/s \Rightarrow e(\infty) = \lim_{s\to 0} s \frac{1}{1+L(s)} \frac{1}{s} = \frac{1}{1+\lim_{s\to 0} L(s)}$$

$$\text{ramp: } tu(t) \to 1/s^2 \Rightarrow e(\infty) = \lim_{s\to 0} s \frac{1}{1+L(s)} \frac{1}{s^2} = \frac{1}{\lim_{s\to 0} s\,L(s)}$$

$$\text{parab: } \tfrac{1}{2} t^2 u(t) \to 1/s^3 \Rightarrow e(\infty) = \lim_{s\to 0} s \frac{1}{1+L(s)} \frac{1}{s^3} = \frac{1}{\lim_{s\to 0} s^2\,L(s)}$$

Position constant: $k_p \triangleq \lim_{s\to 0} L(s)$

Velocity constant: $k_v \triangleq \lim_{s\to 0} s\,L(s)$

Acceleration constant: $k_a \triangleq \lim_{s\to 0} s^2\,L(s)$

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Transfer Function "Type"

A transfer function $G(s)$ of the form

$$L(s) = K\frac{n(s)}{s^k d(s)} \text{ where } s \text{ is not a factor of } d(s) \text{ or } n(s)$$

is said to be of *type* k.

Note that:

$$k_p = \lim_{s\to 0} L(s) = \begin{cases} c \neq 0 & k = 0 \\ \infty & k \geq 1 \end{cases} \qquad k_v = \lim_{s\to 0} s\,L(s) = \begin{cases} 0 & k = 0 \\ c \neq 0 & k = 1 \\ \infty & k \geq 2 \end{cases}$$

$$k_a = \lim_{s\to 0} s^2\,L(s) = \begin{cases} 0 & k = 0,1 \\ c \neq 0 & k = 2 \\ \infty & k \geq 3 \end{cases}$$

Ultimate Error Table ~ Unity Feedback Systems

Input \ Type	0	1	2
$u(t)$	$\frac{1}{1+k_p}$	0	0
$tu(t)$	∞	$\frac{1}{k_v}$	0
$t^2u(t)/2$	∞	∞	$\frac{1}{k_a}$

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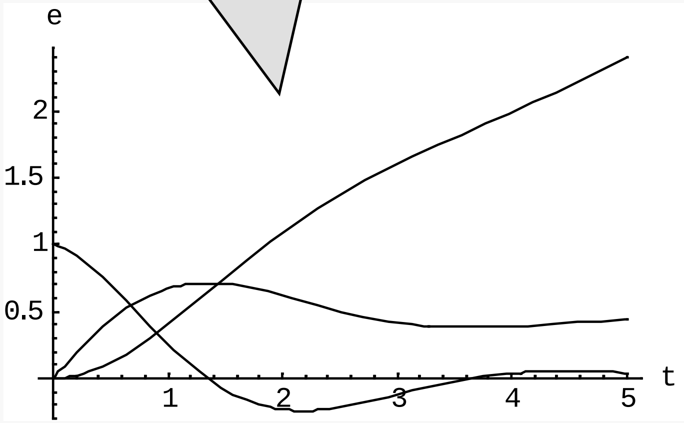
Example

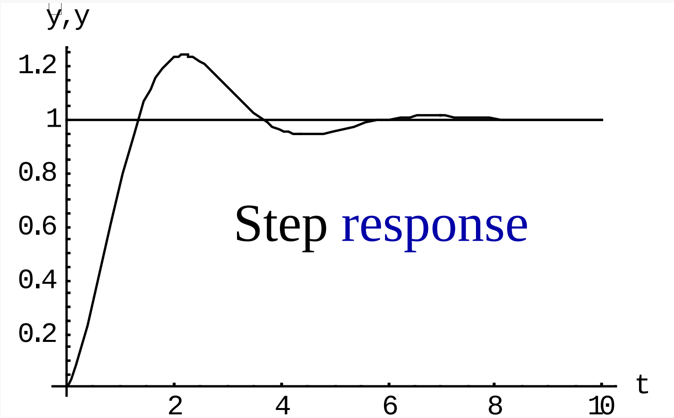
Output responses for standard step, ramp, parabola

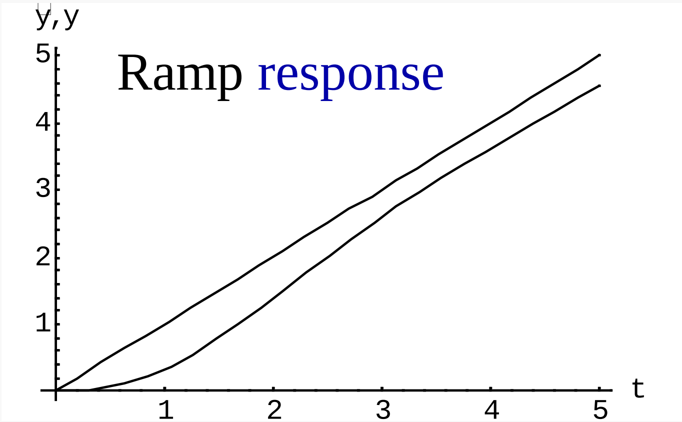
Consider a unity feedback, type 1 system with

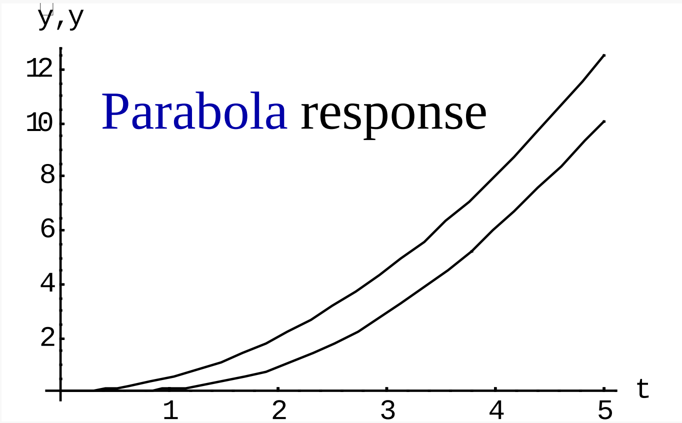
$$L(s)=\frac{1}{4}\left(\frac{s+9}{s}\right)\left(\frac{1}{s+1}\right)$$

Error responses for standard step, ramp, parabola

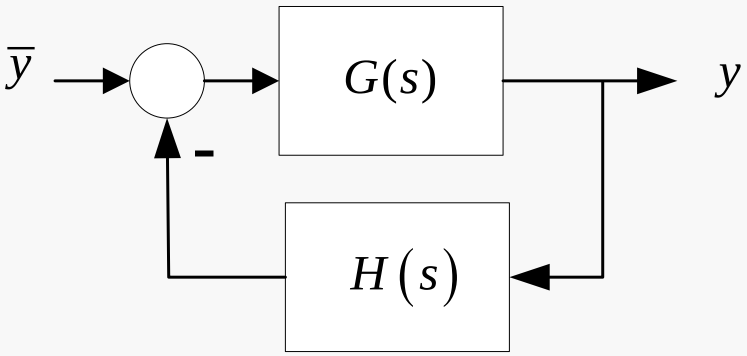








General Case



The closed loop input response transfer function is

$$G_y(s) = \frac{G}{1+GH}$$

The error response transfer function is (recall $e = \bar{y} - y$)

$$G_e(s) = 1 - G_y(s) = \frac{1+GH-G}{1+GH}$$

$$e(\infty) = \lim_{s \to 0} sG_e(s)\bar{Y}(s)$$

Ultimate Error Table ~ General Case

Input \ Type	0	1	2
$u(t)$	$\lim_{s \to 0} G_e(s)$	0	0
$tu(t)$	∞	$\lim_{s \to 0} sG_e(s)$	0
$t^2u(t)/2$	∞	∞	$\lim_{s \to 0} s^2 G_e(s)$

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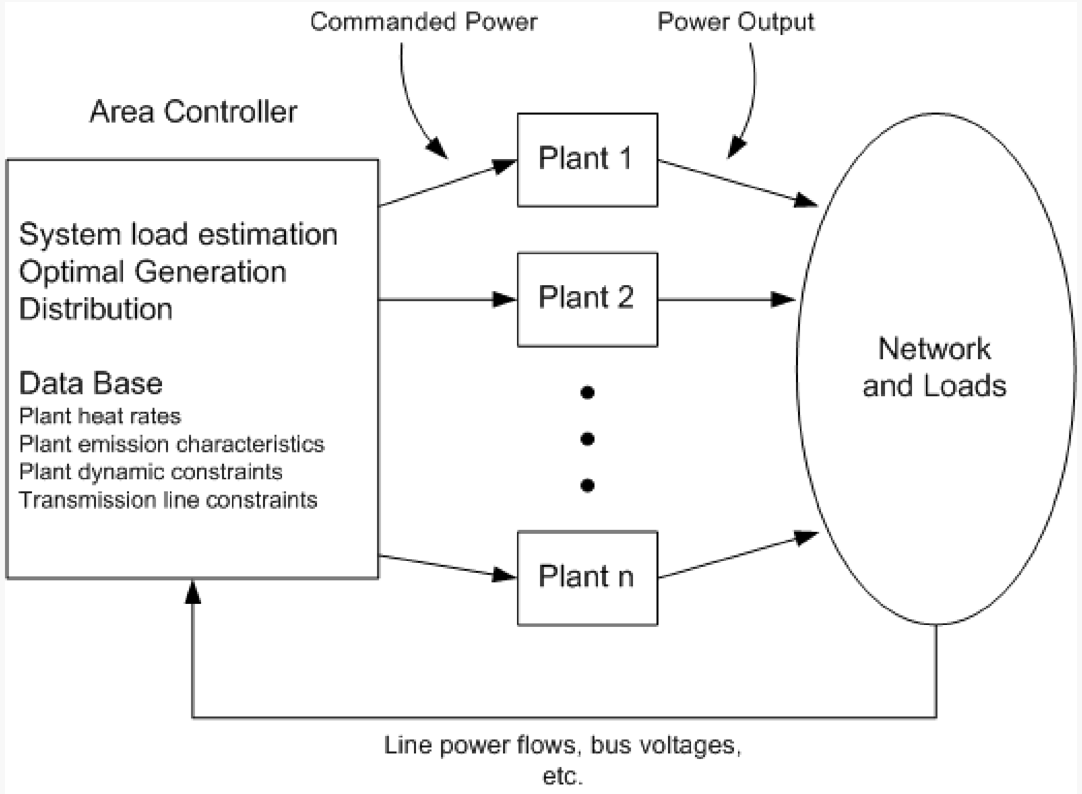
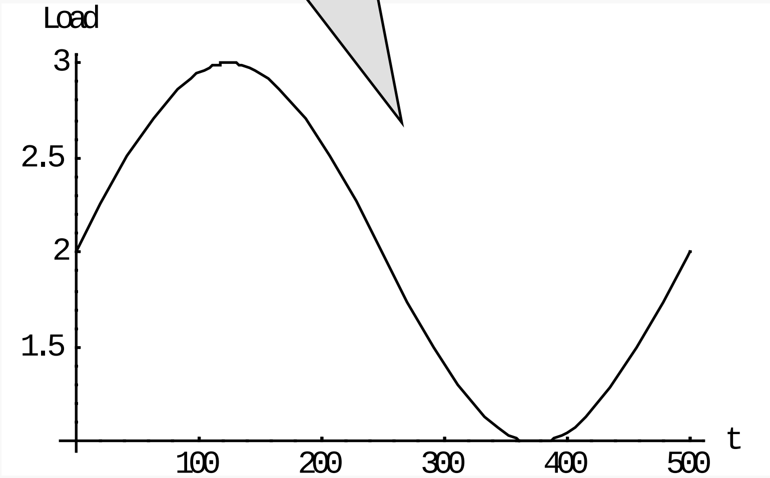
Example: Power System Load Tracking

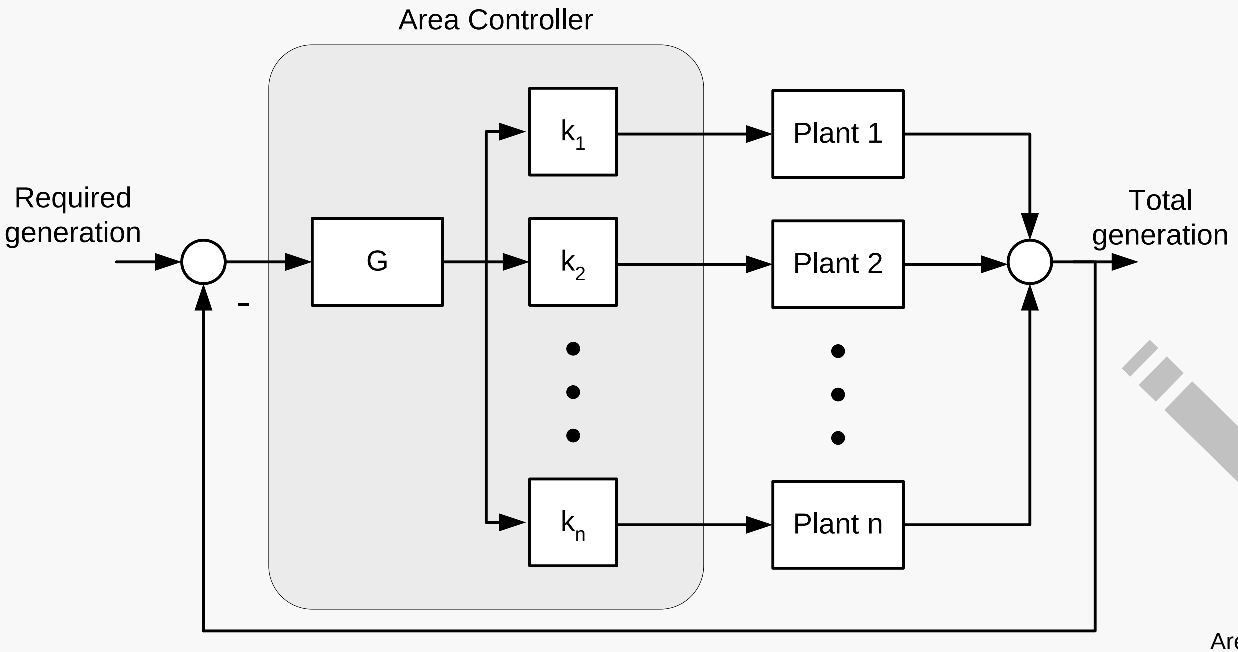


Typical daily load pattern



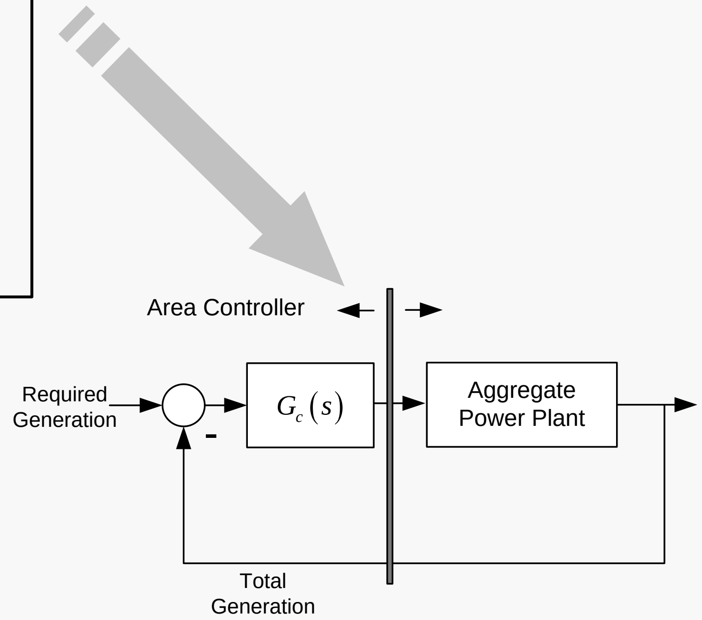
Each power plant that participates in load tracking receives a command of similar shape.

Example, cont’d

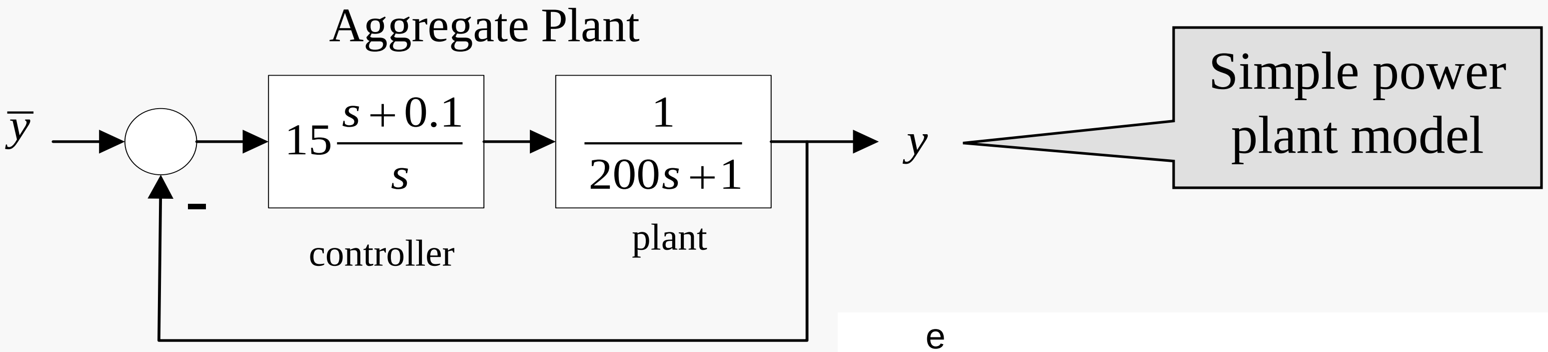


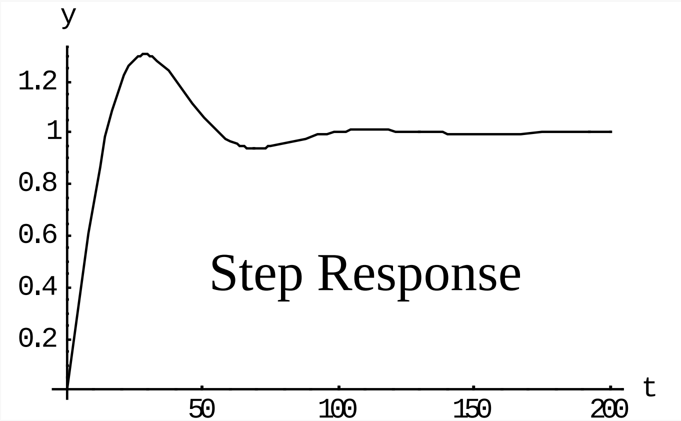
$$k_i, i = 1, 2\ldots, n \quad \text{participation factors}$$

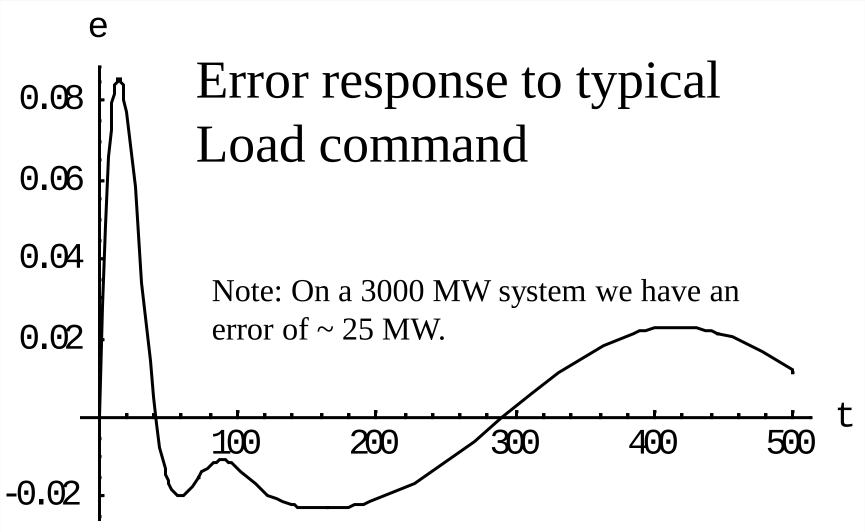
$$k_1 + k_2 + \cdots + k_n = 1$$



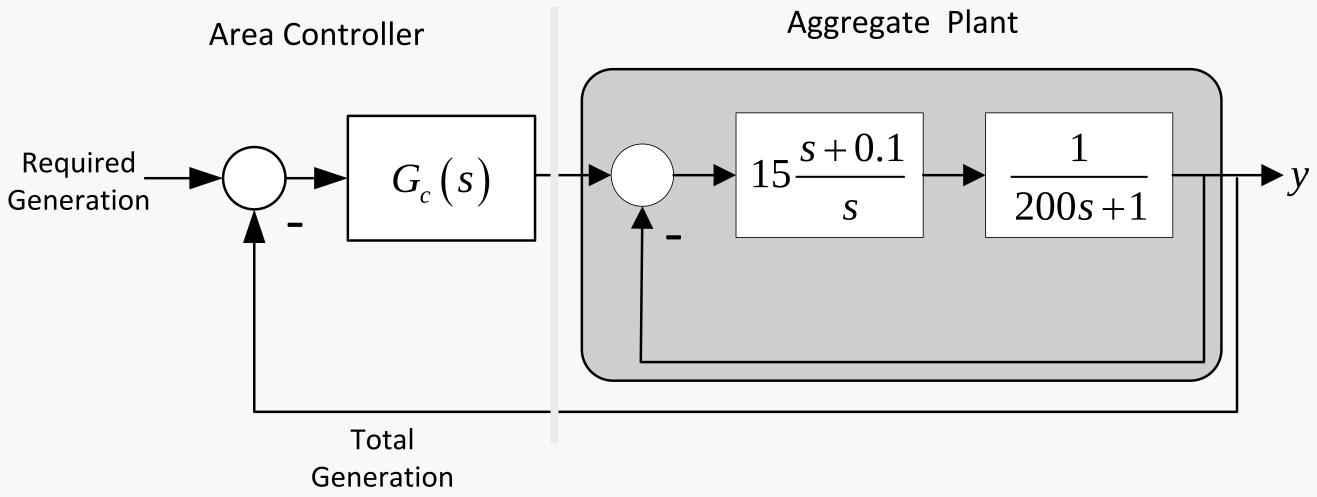
Example: cont'd







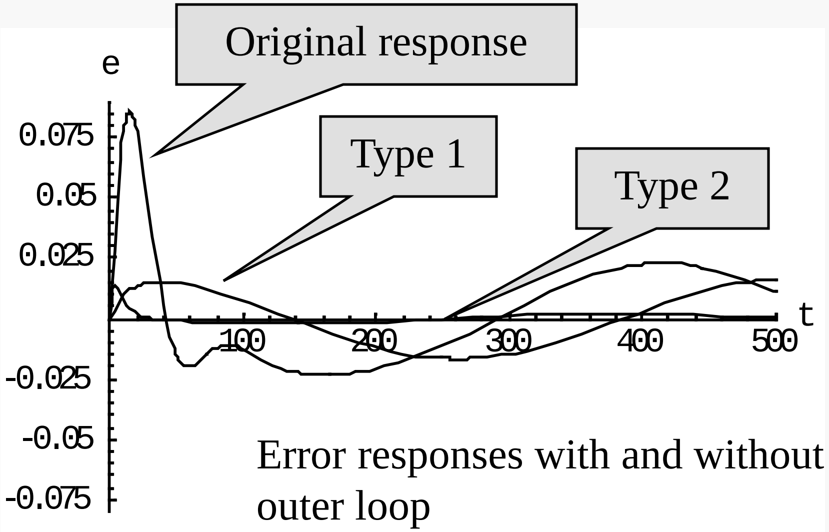
Example: cont'd



Outer Loop Compensator:

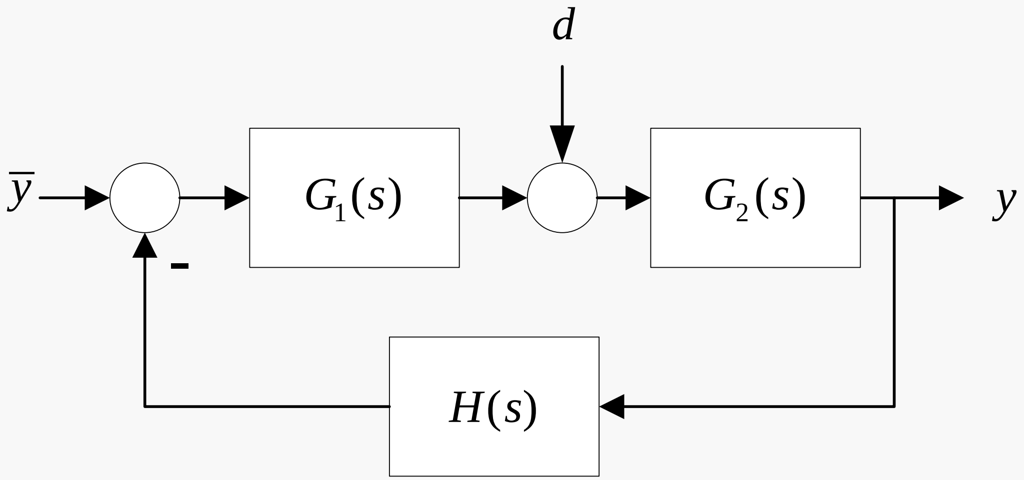
Type 1: $G_c(s) = \frac{200s^2 + 16s + 1.5}{s}$

Type 2: $G_c(s) = \frac{200s^2 + 16s + 1.5}{s^2}$



Error responses with and without outer loop

Ultimate Error Due to Disturbance



$$\text{Transfer Function } d \rightarrow e : G_{ed} = \frac{G_2}{1+G_1G_2H}$$

$$\boxed{e(\infty) = \lim_{s\to 0} sG_{ed}(s)D(s)}$$

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Example: Step Disturbance

$$e(\infty) = \lim_{s \to 0} sG_{ed}(s) D(s)$$

step disturbance: $D(s) = 1/s$

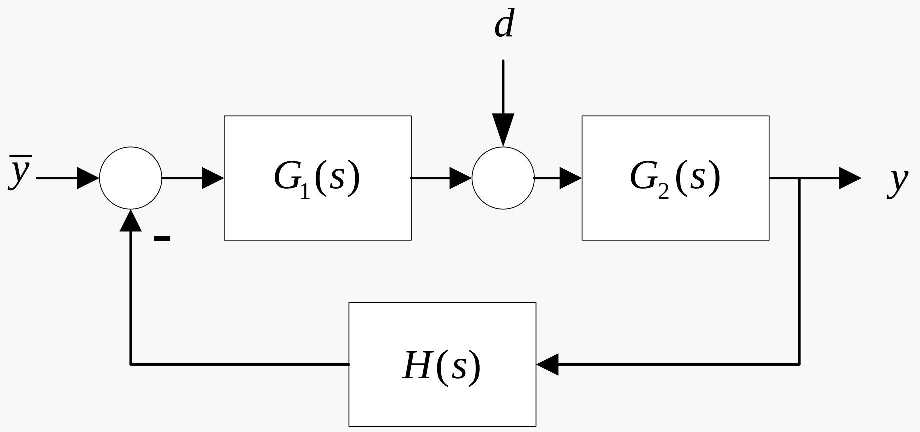
$$e(\infty) = \lim_{s \to 0} \frac{sG_2}{1 + G_1 G_2 H} \frac{1}{s} = \frac{1}{\lim_{s \to 0} \frac{1}{G_2(s)} + \lim_{s \to 0} G_1(s) H(s)}$$

To reduce error:

increase type or gain of $G_1 H$

reduce gain of G_2 (typically not possible)

Example



$$G_2(s) = \frac{1}{s(s+25)},\ H(s) = 1$$

$$G_1(s) = \begin{cases} (a)\text{: P} & 1000 \\ (b)\text{: PI} & 1000\dfrac{(s+2)}{s} \end{cases}$$

First confirm that closed loop is stable!!

Response to command

$$(a)\ G_{\bar{y}e} = \frac{s(s+25)}{s^2+25s+1000},$$

$$e_y(\infty) = \lim_{s\to 0} s\frac{s(s+25)}{s^2+25s+1000}\frac{1}{s} = 0,$$

Response to disturbance

$$G_{de} = \frac{1}{s^2+25s+1000}$$

$$e_d(\infty) = \lim_{s\to 0} s\frac{1}{s^2+25s+1000}\frac{1}{s} = \frac{1}{1000}$$

$$(b)\ G_{\bar{y}e} = \frac{s^2(s+25)}{s^3+25s^2+1000s+2000},$$

$$e_y(\infty) = \lim_{s\to 0} s\frac{s^2(s+25)}{s^3+25s^2+1000s+2000}\frac{1}{s} = 0,$$

$$G_{de} = \frac{s}{s^3+25s^2+1000s+2000}$$

$$e_d(\infty) = \lim_{s\to 0} s\frac{s}{s^3+25s^2+1000s+2000}\frac{1}{s} = 0$$

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Summary

- Concept of “ultimate error,”
- Formulae for unity feedback systems in response to commands,
- Introduction of “error constants,”
- The concept of system “type” and its role in command tracking,
- Formulae for general feedback systems in response to commands and disturbances.

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